# Some elements on system identification from binary measurements of the output

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23/11/2023

# Contents

# 1- The considered identification problem (and some notations)

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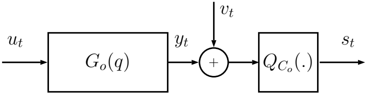
The context

We consider a discrete-time linear system $G_o(q)$ associated to a binary sensor $\mathbf{Q}_{C_o}(.)$




$$\begin{cases} y_t = G_o(q)u_t \\ s_t = \mathbf{Q}_{C_o}(y_t + v_t) \end{cases} \quad (1)$$

- $u_t$ and $y_t$ are the input and output respectively.
- $v_t$ the noise measurement.
- $s_t$ a binary measurement such that:

$$s_t = \mathbf{Q}_{C_o}(y_t + v_t) = \begin{cases} s_{high} & \text{if} \quad y_t + v_t \geq C_o \\ s_{low} & \text{else} \quad y_t + v_t < C_o \end{cases} \quad (2)$$

$C_o$ a constant threshold. For simplicity of presentation we take $C_o = 0$ in the following.

**Objective**

Identification of $G_o(q)$ from $N$ available data $\{u_t, s_t\}_{t=1}^{N}$.

# 1- The considered identification problem (and some notations)

Why this context?

- Economical reason: a low resolution sensor is less expensive than a high resolution sensor.
- Technical reason related to the system to be identified: there is no high-resolution sensor available.
- Technical reason related to the implementation environment: only few binary data can be transmitted or there is not enough memory for high resolution data. on the embedded device.

# 1- The considered identification problem (and some notations)

A short example

- The embedded device: a miniaturized patch that continuously monitors the activity, body orientation and temperature.
- The main use: activity monitoring for frail persons, post-surgery, rehabilitation programs or analysis of circadian rhythm for sleep disorders.
- A binary measurement: in order to save battery life and improve the number of data recordable.

Figure: Example of an embedded device

# 1- The considered identification problem (and some notations)

Literature on the subject


Figure: Approximate number of papers per year

# 1- The considered identification problem (and some notations)

Parametrization

- Only model with finite impulse response considered here: $G_o(q)$ of the form

$$G_o(q) = \sum_{k=0}^{n_b} b_k q^{-k}$$

- The real noise free output expressed as

$$y_t = \sum_{k=0}^{n_b} b_k u_{t-k} = \phi_t^T \theta_o$$

with

$$\theta_o = \begin{pmatrix} \vdots \\ b_i \\ \vdots \end{pmatrix} \quad , \quad \phi_t = \begin{pmatrix} \vdots \\ u_{t-i} \\ \vdots \end{pmatrix} .$$

- $\theta_o \in \mathbb{R}^n$ the parameter vector.

**Objective-bis**

Estimate $\theta_o$ from $N$ available data $\{u_t, s_t\}_{t=1}^{N}$.

# 2- Solution using SVM

# 2- Solution using SVM

Principle

- Consider a FIR system and take $s_{high} = 1$ and $s_{low} = -1$.
- On the figure on the right: in red $\phi_t$ such that $\phi_t^T \theta_o \geq 0$, in blue $\phi_t$ such that $\phi_t^T \theta_o < 0$.
- The boundary satisfies $\phi_t^T \theta_o = 0$.
- The vector $\theta_o$ parameterizes the separating hyperplane in $\mathbb{R}^n$ between $\phi_t$ providing $s_t = 1$ and $\phi_t$ providing $s_t = -1$

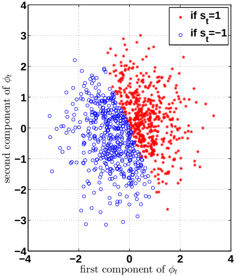



Figure: $\phi_t$ and corresponding binary output $s_t$

$\Downarrow$

The initial identification problem consists in the estimation of the boundary, i.e. it becomes a classification problem.

# 2- Solution using SVM

- The noise can generate some outliers such that some $\phi_t$ are on the wrong side with respect to the separating hyperplane.
- The chosen classification algorithm must provide parametrization of the hyperplane and must take into account possible outliers.

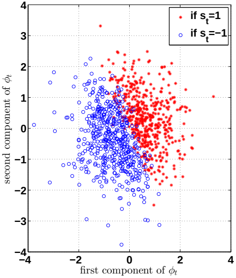



Figure: $\phi_t$ and corresponding binary output $s_t$ (in presence of noise)

$$\Downarrow$$

The vector $\theta_o$ can be estimated using Support Vector Machines (SVM) which use a supervised classification algorithm to estimate parameters of the hyperplane.

## 2- Solution using SVM

Implementation

Estimation of the parameter vector using SVM.

step 1 Compute $\widehat{\theta}$ solution of

$$\begin{aligned} \widehat{\theta} = \quad & \underset{\theta, e}{\operatorname{argmin}} \frac{1}{2} \|\theta\|^2 + \gamma \sum_{t=1}^{N} e_t \\ & \text{s.t} \quad s_t \theta^T \phi_t \geq 1 - e_t \\ & \text{and} \quad e_t \geq 0 \ ; \ \forall t \in [1; N] \end{aligned} \tag{3}$$

step 2 Normalization of $\widehat{\theta}$ with respect to an assumption on $G_o(q)$ (the first component of the impulse response, the static gain, etc.)

PS: the normalization step is necessary to distinguish the solution $\theta^T \phi_t$ from $\gamma \theta^T \phi_t$ with $\gamma > 0$.

## 2- Solution using SVM

Theoretical result[1]

Assumption:

- the input is a symmetrical independent identically distributed process;
- the noise is bounded and white;
- the first parameter of $\theta_o$ is known.

Result: $\widehat{\theta}$ converges almost surely toward $\theta_o$.

[1] G. Li and C. Wen, "Identification of Wiener Systems With Clipped Observations", IEEE Transactions on signal processing, 60(7), 2012

## 2- Solution using SVM

A short example

- We consider the system characterized by

$$G_o(s) = \frac{1}{\left(\frac{s}{p}+1\right)\left(\frac{s^2}{\omega_o^2}+2\zeta\frac{s}{\omega_o}+1\right)}$$

  where $p = 4\pi$, $\omega_o = 20\pi$, $\zeta = 0.2$ and the sampling frequency is $100Hz$.
- The threshold of the binary sensor: $C_o = 0$.
- The input: an independent identically distributed process with uniform distribution in $[-1;1]$.
- The noise: a white gaussian process such that SNR$= 10$dB.
- Number of available data: $N = 2500$.

# 2- Solution using SVM

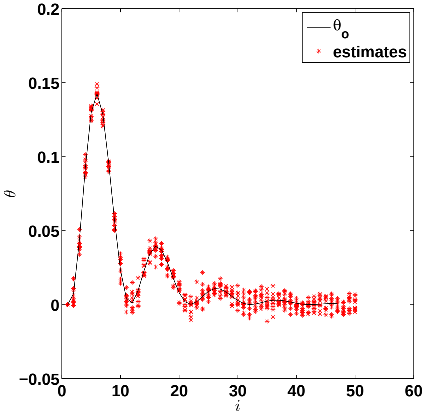



Figure: Estimated parameters

Estimates over 10 realizations of the noise of $n+1=50$ parameters.

# 2- Solution using SVM

Few remarks

+ The noise is taken into account
- Only adapted to the case of FIR system
- Implementation requires sufficient computing capacity
- No real-time implementation
- Not adapted to time variant system

# 3- Solution using a periodic input signal

# 3- Solution using a periodic input signal

Principle

- Consider a FIR system and take $s_{high} = 0$ and $s_{low} = 1$.
- If the input is $n$ periodic, then $y_t$ is $n$ periodic: for all $i \in [0; n-1]$ $y_{i+kn} = y_i$.

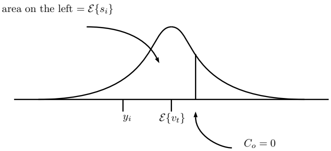



Figure: histogram of $\{y_i + v_{i+kn}\}_k$

- The histogram of $\{y_i + v_{i+kn}\}_k$ is plotted on the figure above and we have

$$\mathcal{E}\{s_i\} = Pr(y_i + v_{i+kn} < 0) = Pr(v_{i+kn} < -y_i) \quad (4)$$

- Denote $F_v(.)$ the cumulative distribution function of $\{v_t\}$, $\mathcal{E}\{s_i\}$ satisfies

$$\mathcal{E}\{s_i\} = F_v(-y_i)$$
$$y_i = -F_v^{-1}(\mathcal{E}\{s_i\}) \quad (5)$$

# 3- Solution using a periodic input signal

From the above it is possible to estimate the high resolution output $y_i$ for $i \in [0; n-1]$, then

$$N(\{y_i\}_i) = M(\{u_i\}_i)\theta_o \tag{6}$$

with

$$N(\{y_i\}_i) = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$M(\{u_i\}_i) = \begin{pmatrix} u_0 & u_1 & \cdots u_{n-1} & \\ u_1 & u_2 & \cdots \vdots & \\ \vdots & \ddots & \ddots & \vdots \\ u_{n-1} & u_0 & \cdots u_{n-2} & \end{pmatrix}$$

$$\Downarrow$$

Once we have estimated the high resolution output $\{y_i\}$ we can estimate the parameter vector.

# 3- Solution using a periodic input signal

Implementation

Estimation of the parameter vector through the estimation of $\{y_i\}_{i=0}^{n-1}$.

step 1 Estimate $\mathcal{E}\{s_i\}$ for $i \in \{0, 1, ..., n-1\}$

$$\widehat{\mathcal{E}\{s_i\}} = \frac{1}{N_n} \sum_{k=1}^{N_n} s_{i+k*n} \quad (7)$$

with $N_n = \lfloor \frac{N}{n} \rfloor$ the integer portion of $\frac{N}{n}$.

step 2 Compute $\widehat{y}_i$ for $i \in \{0, 1, ..., n-1\}$

$$\widehat{y}_i = -F_v^{-1}\left(\widehat{\mathcal{E}\{s_i\}}\right) \quad (8)$$

step 3 Compute $\widehat{\theta}$ using

$$\widehat{\theta} = M(\{u_i\}_i)^{-1} N(\{\widehat{y}_i\}_i) \quad (9)$$

# 3- Solution using a periodic input signal

Theoretical result[2]

Assumption:

- the input is $n$ periodic such that $M$ is full rank;
- $\{v_t\}$ is a sequence of i.i.d random variables whose distribution function $F_v(.)$ and its inverse $F_v(.)^{-1}$ are continuous and known;

Result: $\widehat{\theta}$ converges almost surely toward $\theta_o$.

[2] L.Y. Wang, G.G. Yin and J. Zhang, "Joint identification of plant rational models and noise distribution functions using binary-valued observations", Automatica, 42, 2006

# 3- Solution using a periodic input signal

The same short example than previously

- We consider the system characterized by

$$G_o(s) = \frac{1}{\left(\frac{s}{p}+1\right)\left(\frac{s^2}{\omega_o^2}+2\zeta\frac{s}{\omega_o}+1\right)}$$

where $p = 4\pi$, $\omega_o = 20\pi$, $\zeta = 0.2$ and the sampling frequency is $100Hz$.
- The threshold of the binary sensor: $C_o = 0$.
- The input: a periodic impulse sequence (with period $= n+1$).
- The noise: a white gaussian process such that SNR$= -10$dB.
- Number of available data: $N = 2500$.

# 3- Solution using a periodic input signal

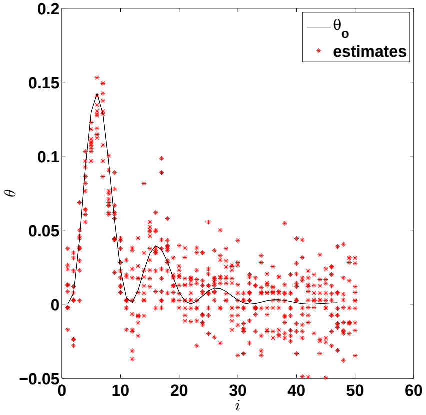



Figure: Estimated parameters

Estimates over 10 realizations of the noise of $n+1=50$ parameters.

# 3- Solution using a periodic input signal

Few remarks

+ Extension for IIR system
+ Real-time implementation
+ Adapted to time variant system
- Hard constraint on the input
- Hard constraint on the knowledge of the noise

# 4- Solution using a correlation analysis

# 4- Solution using a correlation analysis

## Principle

- We take $s_{high} = 1$ and $s_{low} = 0$.
- If the input and the noise are gaussian process, then it can be shown that there exists $\gamma > 0$ such that for all $i$ we have

$$\mathcal{E}\{s_t u_{t-i}\} = \gamma \mathcal{E}\{y_t u_{t-i}\} \quad (10)$$

- Denote $\{h_i\}$ the impulse response, from the fact that $y_t = \sum_{k=0}^{\infty} h_k u_{t-k} + v_t$ we get

$$\mathcal{E}\{s_t u_{t-i}\} = \gamma \sum_{k=0}^{\infty} h_k \mathcal{E}\{u_{t-k} u_{t-i}\} \quad (11)$$

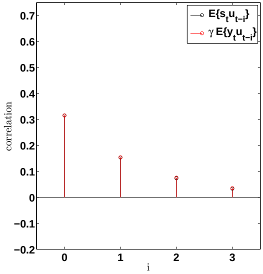



Figure: $\mathcal{E}\{s_t u_{t-i}\}$ and $\mathcal{E}\{y_t u_{t-i}\}$ estimated using $10^4$ data (if you see only one color it's a good news).

# 4- Solution using a correlation analysis

From the above, if we consider a FIR system, then

$$N(\{\mathcal{E}\{s_t u_{t-i}\}\}_i) = \gamma M(\{\mathcal{E}\{u_t u_{t-i}\}\}_i)\theta_o \qquad (12)$$

with

$$N(\{\mathcal{E}\{s_t u_{t-i}\}\}_i) = \begin{pmatrix} \mathcal{E}\{s_t u_t\} \\ \mathcal{E}\{s_t u_{t-1}\} \\ \vdots \\ \mathcal{E}\{s_t u_{t-n}\} \end{pmatrix}$$

$$M(\{\mathcal{E}\{u_t u_{t-i}\}\}_i) = \begin{pmatrix} \mathcal{E}\{u_t u_t\} & \mathcal{E}\{u_t u_{t-1}\} & \cdots & \mathcal{E}\{u_t u_{t-n}\} \\ \mathcal{E}\{u_t u_{t-1}\} & \mathcal{E}\{u_t u_t\} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \mathcal{E}\{u_t u_{t-n}\} & \cdots & \cdots & \mathcal{E}\{u_t u_t\} \end{pmatrix}$$

$$\Downarrow$$

The initial identification problem corresponds to an identification problem with the Wiener-Hopf equation.

# 4- Solution using a correlation analysis

Implementation

Estimation of the parameter vector through the estimation of the correlation between $s_t$ and $u_t$.

step 1 Estimate $\mathcal{E}\{s_t u_{t-i}\}$ and $\mathcal{E}\{u_t u_{t-i}\}$ for $i \in \{0, 1, ..., n-1\}$

$$\mathcal{E}\{\widehat{s_t u_{t-i}}\} = \frac{1}{N-i} \sum_{k=1+i}^{N} s_k u_{k-i} \tag{13}$$

$$\mathcal{E}\{\widehat{u_t u_{t-i}}\} = \frac{1}{N-i} \sum_{k=1+i}^{N} u_k u_{k-i} \tag{14}$$

step 2 Compute $\widehat{\theta}$ using

$$\widehat{\theta} = M(\{\mathcal{E}\{\widehat{u_t u_{t-i}}\}\}_i)^{-1} N(\{\mathcal{E}\{\widehat{s_t u_{t-i}}\}\}_i) \tag{15}$$

step 3 Normalization of $\widehat{\theta}$ with respect to an assumption on $G_o(q)$ (the first component of the impulse response, the static gain, etc.)

# 4- Solution using a correlation analysis

Theoretical result[3]

Assumption:

- the input is a zero mean white gaussian process;
- the noise is a zero mean white gaussian process;
- the first parameter of $\theta_o$ is known.

Result: $\widehat{\theta}$ converges almost surely toward $\theta_o$.

Extended result in the case of a colored gaussian process on the input.

[3] Q. Song, "Recursive identification of systems with binary-valued outputs and with ARMA noises", Automatica, 93, 2018

# 4- Solution using a correlation analysis

The same short example than previously

- We consider the system characterized by

$$G_o(s) = \frac{1}{\left(\frac{s}{p}+1\right)\left(\frac{s^2}{\omega_o^2}+2\zeta\frac{s}{\omega_o}+1\right)}$$

  where $p = 4\pi$, $\omega_o = 20\pi$, $\zeta = 0.2$ and the sampling frequency is $100Hz$.
- The threshold of the binary sensor: $C_o = 0$.
- The input: a white gaussian process.
- The noise: a white gaussian process such that SNR$=$ 10dB.
- Number of available data: $N = 2500$.

# 4- Solution using a correlation analysis

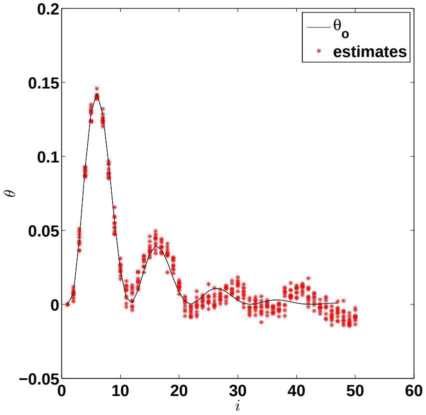



Figure: Estimated parameters

Estimates over 10 realizations of the noise of $n + 1 = 50$ parameters.

# 4- Solution using a correlation analysis

Few remarks

+ The noise is taken into account

+ Real-time implementation

+ Adapted to time variant system

+- Possible extension to IIR model (if the input is white)

- Hard constraint on the input

# 5- Solution using a set membership identification formulation

- 1- The considered identification problem (and some notations)
- 2- Solution using SVM
- 3- Solution using a periodic input signal
- 4- Solution using a correlation analysis
- **5- Solution using a set membership identification formulation**
- 6- Solution using a gradient based algorithm
- 7- A short application
- 8- Some conclusions

# 5- Solution using a set membership identification formulation

Principle

- Consider a FIR system, take $s_{high} = 1$ and $s_{low} = -1$ and $v_t = 0$.
- $y_t$ and $s_t$ have the same sign, there exists $\gamma > 0$ such that

$$\begin{cases} \text{when } s_t = 1, \text{ then } 0 < \gamma y_t < 2 \\ \text{when } s_t = -1, \text{ then } -2 < \gamma y_t < 0 \end{cases}$$

- There exists $\gamma > 0$ such that $s_t$ can be formulated as

$$s_t = \gamma y_t + b_t = \phi_t^T \gamma \theta_o + b_t \quad (16)$$

with $|b_t| < 1$.

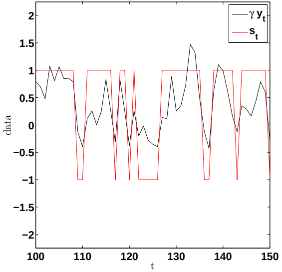



Figure: $s_t$ and $\gamma y_t$.

$$\Downarrow$$

The initial identification problem corresponds to identification problem in presence of bounded noise.

# 5- Solution using a set membership identification formulation

Implementation

Use an Ellipsoidal Outer Bounding (EOB) algorithm ensuring the estimation of a parameter vector such that $|s_t - \phi_t^T \widehat{\overline{\theta}}| < 1$.

step 1 Compute recursively $\widehat{\overline{\theta}}_t$ with

$$\left\{ \begin{array}{l} \epsilon_{t/t-1} = s_t - \phi_t^T \widehat{\overline{\theta}}_{t-1} \\ \Gamma_t = \frac{P_{t-1}\phi_t \sigma_t}{\lambda + \phi_t^T P_{t-1}\phi_t \sigma_t} \\ \widehat{\overline{\theta}}_t = \widehat{\overline{\theta}}_{t-1} + \Gamma_t \epsilon_{t/t-1} \\ P_t = (I_{n+1} - \Gamma_t \phi_t^T) \frac{P_{t-1}}{\lambda} \end{array} \right. \quad (17)$$

with $0 < \lambda < 1$ (forgetting factor) and $\sigma_t$ designed such that $|s_t - \phi_t^T \widehat{\overline{\theta}}_t| < 1$.

step 2 Normalization of $\widehat{\theta}$ with respect to an assumption on $G_o(q)$ (the first component of the impulse response, the static gain, etc.)

$$\widehat{\theta}_t = \frac{1}{\gamma} \widehat{\overline{\theta}}_t$$

# 5- Solution using a set membership identification formulation

Theoretical result[4]

Assumption:

- while $\{\sqrt{\sigma_t}\phi_t\}$ is a persistently exciting sequence;
- there is no noise;
- the norm of $\theta_o$ is known.

Result: there exists $\alpha$ such that $\|\theta_o - \widehat{\theta}_t\|_2 \leq \alpha\lambda^t\|\theta_o - \widehat{\theta}_0\|_2$.

[4] M. Pouliquen, T. Menard, E. Pigeon, O. Gehan and A. Goudjil, "Recursive System Identification Algorithm using Binary Measurements", ECC, 2016

# 5- Solution using a set membership identification formulation

Few remarks

+ Extension for IIR system
+ Real-time implementation
+ Adapted to time variant system
- The noise is not taken into account

# 5- Solution using a set membership identification formulation

The same short example than previously

- We consider the system characterized by

$$G_o(s) = \frac{1}{\left(\frac{s}{p}+1\right)\left(\frac{s^2}{\omega_o^2}+2\zeta\frac{s}{\omega_o}+1\right)}$$

  where $p = 4\pi$, $\omega_o = 20\pi$, $\zeta = 0.2$ and the sampling frequency is $100Hz$.
- The threshold of the binary sensor: $C_o = 0$.
- The input: an independent identically distributed process with uniform distribution in $[-1; 1]$.
- The noise: a white gaussian process such that SNR$= 10$dB.
- Number of available data: $N = 2500$.

# 5- Solution using a set membership identification formulation

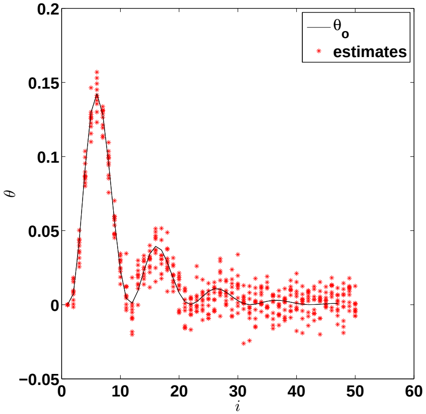



Figure: Estimated parameters

Estimates over 10 realizations of the noise of $n + 1 = 50$ parameters.

# 6- Solution using a gradient based algorithm

# 6- Solution using a gradient based algorithm

Principle

- Consider a FIR system, take $s_{high} = 1$ and $s_{low} = -1$ and $v_t = 0$.
- Let consider the cost function (plotted on the right for two parameters)

$$J(\theta) = \frac{1}{N}\sum_{t=1}^{N}(s_t - \widehat{s}_t)^2 \widehat{y}_t^2 \quad (18)$$

- It is shown that: (1) $J(\theta)$ has infinitely many minima of the form $\gamma\theta_o$, (2) $J(\theta)$ is convex in a neighborhood of its minima.

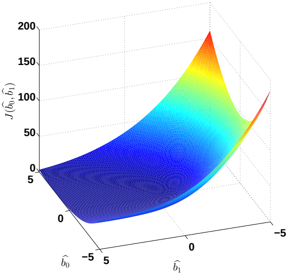



Figure: $J(\widehat{\theta})$ as a function of $\widehat{\theta}$.

$\Downarrow$

It seems possible to estimate $\theta_o$ using a classical LMS approach of the form

$$\widehat{\widehat{\theta}}_t = \widehat{\widehat{\theta}}_{t-1} - \alpha_t \frac{dJ_t}{d\theta}_{\theta=\widehat{\widehat{\theta}}_{t-1}} \text{ with } J_t = (s_t - \widehat{s}_t)^2 \widehat{y}_t.$$

# 6- Solution using a gradient based algorithm

Implementation

Use of a LMS approach including a normalization step.

step 1 Compute recursively $\hat{\hat{\theta}}_t$ with

$$\left\{ \begin{array}{l} \hat{y}_{t/t-1} = \phi_t^T \hat{\theta}_{t-1} \\ \epsilon_{t/t-1} = |s_t - Q_0(\hat{y}_{t/t-1})| \hat{y}_{t/t-1} \\ \alpha_t = \frac{1}{\phi_t^T \phi_t} \\ \hat{\hat{\theta}}_t = \hat{\theta}_{t-1} - \mu \alpha_t \phi_t \epsilon_{t/t-1} \end{array} \right. \tag{19}$$

step 2 Normalization (here normalization so as to have $\|\hat{\theta}_t\|_2 = 1$)

$$\hat{\theta}_t = \frac{1}{\gamma_t} \hat{\hat{\theta}}_t \tag{20}$$

with $\gamma_t = \sqrt{1 - 2\mu(1-\mu)\alpha_t |s_t - Q_0(\hat{y}_{t/t-1})| \hat{y}^2_{t/t-1}}$

# 6- Solution using a gradient based algorithm

Theoretical result[5]

Assumption:

- the input signal is a sequence of i.i.d. with a uniform distribution;
- there is no noise;
- the norm of $\theta_o$ is known ($\|\theta_o\|_2 = 1$ previously).

Result: $\widehat{\theta}_t$ converges almost surely toward $\theta_o$.

[5] K. Jafari, J. Juillard and M. Roger, "Convergence analysis of an online approach to parameter estimation problems based on binary observations", Automatica, 48(11), 2012

# 6- Solution using a gradient based algorithm

The same short example than previously

- We consider the system characterized by

$$G_o(s) = \frac{1}{\left(\frac{s}{p}+1\right)\left(\frac{s^2}{\omega_o^2}+2\zeta\frac{s}{\omega_o}+1\right)}$$

  where $p = 4\pi$, $\omega_o = 20\pi$, $\zeta = 0.2$ and the sampling frequency is $100Hz$.
- The threshold of the binary sensor: $C_o = 0$.
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# 6- Solution using a gradient based algorithm

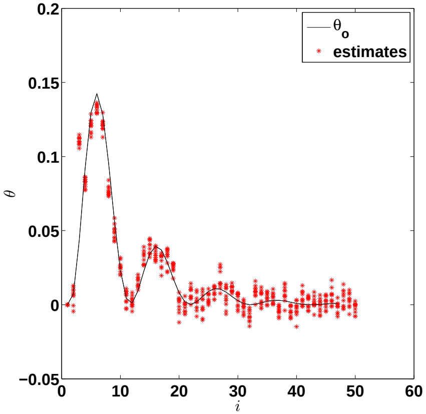


Figure: Estimated parameters

Estimates over 10 realizations of the noise of $n + 1 = 50$ parameters.

# 6- Solution using a gradient based algorithm

Few remarks

+ Real-time implementation
+ Adapted to time variant system
- The noise is not taken into account
- Only adapted to the case of FIR system

# 7- A short application

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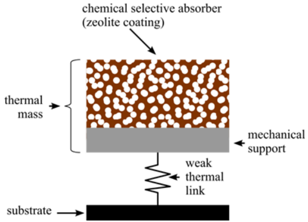
The device: a gas sensor

- The device can be modeled by a 1st order thermal system with time constant $\tau_{th}$.

$$trft\ fct = \frac{K}{1 + \tau_{th} s} \qquad (21)$$

- The thermal time constant $\tau_{th} = \frac{C_{th}}{G_{th}}$ depends on the thermal mass $C_{th}$.

- The thermal mass $C_{th}$ depends on the quantity of adsorbed gas and consequently on the gas concentration.

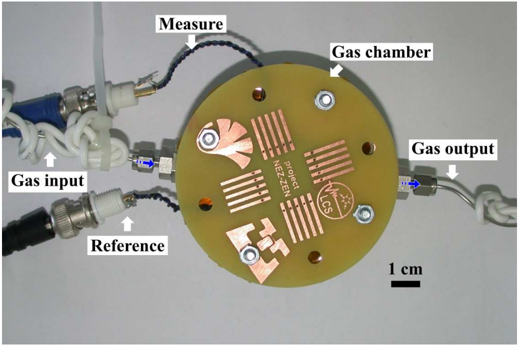



⇓

Estimation of the gas concentration variation through an estimation of the time constant variation.

# 7- A short application

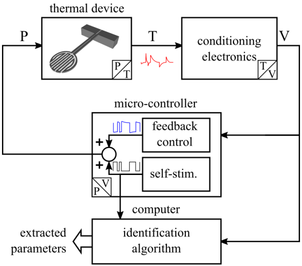
Picture of the sensor cell



# 7- A short application

The initial operation mode

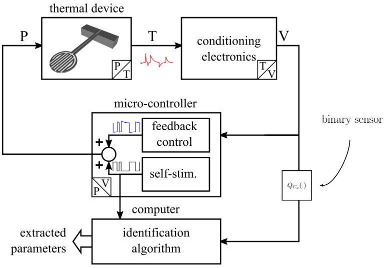
- The thermal device is operated in closed-loop mode using a proportional-integral (PI) controller.
- The PI controller is set to keep constant the temperature of the thermal device.
- An external sequence is applied to stimulate the thermal device.
- An identification algorithm realizes the estimation of the thermal parameters $\tau_{th}$.



# 7- A short application

A variant on the operation mode

- A binary sensor is added.
- The closed loop still operates (to keep constant the temperature).
- Our identification strategy:
  1. Identify the impulse response of the closed loop from the external sequence and the binary output.
  2. Compute the equivalent 2nd order transfer function of the closed loop.
  3. Extract the time constant $\tau_{th}$ from the denominator of the closed loop (not difficult to extract).



# 7- A short application

Experimental conditions.

- An experiment with various concentrations of water vapor: successively 0ppm, 5000ppm, 0ppm, 10000ppm and 0ppm.
- The duration of the experiment was 100 minutes.
- The sampling period was equal to 5ms.
- Identification on time interval of 120s.
- Comparison with results obtained with a closed loop identification algorithm using the high resolution output: Closed Loop Simplified Refined Instrumental Variable Continuous-time CLSRIVC [6].

PS: The algorithm used for the estimation of the impulse response of the closed loop from binary data was the solution using a set membership identification formulation (section 5).

[6] M. Gilson, H. Garnier, P. Young and P. Van den Hof, "Instrumental variable methods for closed-loop continuous-time system identification", in Identification of Continuous-Time Models from Sampled Data, H. Garnier, L. Wangs Springer Verlag, London, 2008

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Results

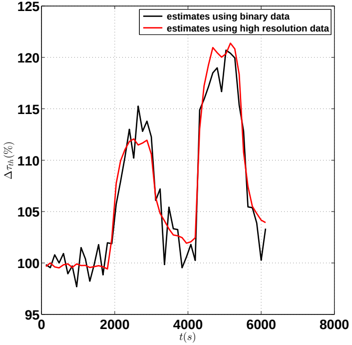


Figure: Relative variation of the time constant $\tau_{th}$

# 8- Some conclusions

# 8- Conclusions

Some concluding remarks

- Several available solutions, your choice may depend on your implementation framework (real time, design of the input, etc.).
- Implementations in real conditions provide coherent results.
- A complementary information is always required: knowledge of the static gain, knowledge of the threshold if different from 0, knowledge of the noise characteristics, etc.
- There are some other methods dedicated to the identification of FIR system.

# 8- Some conclusions

Some variants

- Time series modeling and analysis.
- Continuous time system identification;
- Solutions for estimation of state-space model;
- Identification from binary data on the input and the output.

# 8- Some conclusions

Thank you!

# Annexe 1

Annexe 1: a short numerical example

We consider the following simulation conditions

- $G_o(q)$ such that $G_o(z) = 1 + 0.5z^{-1}$;
- $\{u_t\}$= a white gaussian noise;
- no noise: $v_t = 0$;
- the binary sensor such that $s_t = \mathbf{Q}_{C_o=0}(y_t) = \begin{cases} 1 & \text{if} \quad y_t \geq 0 \\ -1 & \text{else} \quad y_t < 0 \end{cases}$.

We consider a model with the same structure, i.e. $\widehat{G}(z) = \widehat{b_0} + \widehat{b_1} z^{-1}$, and we compute the quadratic cost function

$$J(\widehat{b_0}, \widehat{b_1}) = \frac{1}{N} \sum_{t=1}^{N} (s_t - \widehat{s}_t)^2$$

(with $\widehat{s}_t = \mathbf{Q}_0(\widehat{y}_t)$) for different values of $\widehat{b_0}$ and $\widehat{b_1}$.

# Annexe 1

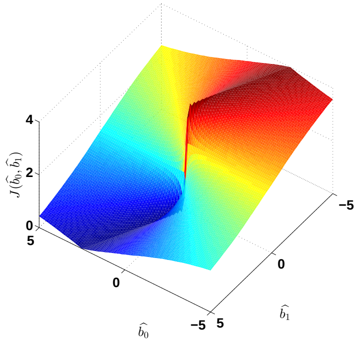


Figure: $J(\widehat{b_0}, \widehat{b_1})$ as a function of $\widehat{b_0}$ and $\widehat{b_1}$

The quadratic cost function seems a bit exotic and the estimation of parameters is not realized by directly minimizing $\frac{1}{N}\sum_{t=1}^{N}(s_t - \widehat{s}_t)^2$.