

Stability of high frequency amplifiers via
singularities of the Harmonic Transfer Function
Workshop ERNSI

L.BARATCHART, S.FUEYO and J-B.POMET

27 May 2021

Inria

Explaining the title

- Dynamical system with periodic solution
Is it locally stable?

Explaining the title

- Dynamical system with periodic solution
Is it locally stable?
- Delay or 1-D Hyperbolic PDE.

Explaining the title

- Dynamical system with periodic solution
Is it locally stable?
- Delay or 1-D Hyperbolic PDE.
- Characterization in the frequency domain.

Explaining the title

- Dynamical system with periodic solution
Is it locally stable?
- Delay or 1-D Hyperbolic PDE.
- Characterization in the frequency domain.
- Nonlinear electric circuit containing transmission lines.

Summary

- 1 Introduction
 - General context
- 2 Circuits without transmission lines
- 3 General circuits containing lossless transmission lines

1 Introduction

- General context

2 Circuits without transmission lines

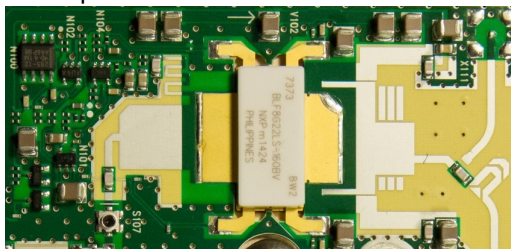
3 General circuits containing lossless transmission lines

An amplifier is made of interconnected

- 1 resistors, inductors, capacitors,
- 2 diodes/transistors (active component),
- 3 lossless transmission lines inducing delays.

Forcing periodic input (large signal functioning)

| | periodic solution in the amplifier.



Motivation

- Amplifiers at high frequency are ubiquitous (Cell phones, relays...). They need to be quick to design.
- Required Computer-assisted design (CAD) and simulation.
- time domain inefficient Need to work in frequency domain.
- “frequency simulation” tools give a reliable prediction of the periodic solution, but that solution might be unstable price to pay to work in the frequency domain.
- **Need** for a tool to predict stability/unstability from the frequency domain data.

Harmonic Balance: a numerical heuristic

The Harmonic Balance method, through Fourier development, Laplace transform and fixed point methods permits to :

- approximate a periodic solution of the circuit,
- linearize the (equations of) circuit around the periodic solution,
- compute the frequency response to a small periodic signal which disturbs the linearized circuit.

Harmonic balance method : Numerically implemented. (ADS, MWO and Spectre softwares)



Stability of the periodic solution

|| Frequency response of the linearized system to a perturbation signal: input-output system

Stability of the periodic solution

| | Frequency response of the linearized system to a perturbation signal: input-output system

| | if the input is $e^{i\omega t}$ then the output is $G(i\omega, t)e^{i\omega t}$

Stability of the periodic solution

- | | Frequency response of the linearized system to a perturbation signal: input-output system
- | | if the input is $e^{i\omega t}$ then the output is $G(i\omega, t)e^{i\omega t}$
- | | Rational approximation tools used on the Fourier development of $G(i\omega, \cdot)$ (Harmonic Transfer Function)

Stability of the periodic solution

- | | Frequency response of the linearized system to a perturbation signal: input-output system
- | | if the input is $e^{i\omega t}$ then the output is $G(i\omega, t)e^{i\omega t}$
- | | Rational approximation tools used on the Fourier development of $G(i\omega, \cdot)$ (Harmonic Transfer Function)
- | | Implementation STAN tool software or see method in the paper [CSO⁺18].

Stability of the periodic solution

- | | Frequency response of the linearized system to a perturbation signal: input-output system
- | | if the input is $e^{i\omega t}$ then the output is $G(i\omega, t)e^{i\omega t}$
- | | Rational approximation tools used on the Fourier development of $G(i\omega, \cdot)$ (Harmonic Transfer Function)
- | | Implementation STAN tool software or see method in the paper [CSO⁺18].

Our focus in this talk:

- Structure of the **Harmonic Transfer Function**: analytic operator valued.
- its singularities, are they poles?
- links with local stability in the time domain.

1 Introduction

- General context

2 Circuits without transmission lines

3 General circuits containing lossless transmission lines

No transmissions line. Equations

Resistors, transistors, inductors, capacitors and forcing periodic
signal

+

Kirshhoff's laws.

Assumption: Existence of a T -periodic solution.

No transmissions line. Equations

Resistors, transistors, inductors, capacitors and forcing periodic signal

+

Kirshhoff's laws.

Assumption: Existence of a T -periodic solution.

Linearization around periodic solution:

$$\frac{dx(t)}{dt} = A(t)x(t),$$

x : state of the circuit (currents or voltages),

No transmissions line. Equations

Resistors, transistors, inductors, capacitors and forcing periodic signal

+

Kirshhoff's laws.

Assumption: Existence of a T -periodic solution.

Linearization around periodic solution:

$$\frac{dx(t)}{dt} = A(t)x(t),$$

x : state of the circuit (currents or voltages),

Paradigm: Exponential stability of the origin of the linearized equation implies local stability of the periodic solution.

No transmission lines. Time-invariant case

Dynamical system:

$$\frac{dx(t)}{dt} = Ax(t),$$

x : state of the circuit (current or voltage).

No transmission lines. Time-invariant case

Dynamical system:

$$\frac{dx(t)}{dt} = Ax(t),$$

x : state of the circuit (current or voltage).

Definition

The origin of the system is exponentially stable if there exists γ, K positive such that:

$$\|x(t)\| \leq Ke^{-\gamma t} \|x(0)\|,$$

$x(\cdot)$ solution of the system.

No transmission lines. Time-invariant case

Dynamical system:

$$\frac{dx(t)}{dt} = Ax(t),$$

x : state of the circuit (current or voltage).

Definition

The origin of the system is exponentially stable if there exists γ, K positive such that:

$$\|x(t)\| \leq Ke^{-\gamma t} \|x(0)\|,$$

$x(\cdot)$ solution of the system.

Theorem (classical)

Origin exp. stable if and only if the real part of the eigenvalues of A are strictly negative.

Time-invariant case

Input-output system :

$$\begin{cases} \frac{d}{dt}y(t) = Ay(t) + Bu(t) \\ z(t) = Cy(t) + Du(t) \end{cases}, t \geq 0.$$

Time-invariant case

Input-output system :

$$\begin{cases} \frac{d}{dt}y(t) = Ay(t) + Bu(t) \\ z(t) = Cy(t) + Du(t), t \geq 0. \end{cases}$$

Transfer function : $H(s) := C(sI_n - A)^{-1}B + D$.

It satisfies the property: $Z(s) = H(s)U(s)$,

$Z(\cdot)$, $U(\cdot)$ Laplace transform of $z(\cdot)$, $u(\cdot)$.

Time-invariant case

Input-output system :

$$\begin{cases} \frac{d}{dt}y(t) = Ay(t) + Bu(t) \\ z(t) = Cy(t) + Du(t) \end{cases}, t \geq 0.$$

Transfer function : $H(s) := C(sI_n - A)^{-1}B + D$.

It satisfies the property: $Z(s) = H(s)U(s)$,

$Z(\cdot)$, $U(\cdot)$ Laplace transform of $z(\cdot)$, $u(\cdot)$.

Theorem (classical)

If the system is observable and controllable, the singularities of the transfer function are poles and are the eigenvalues of A . Therefore the system is exponentially stable if and only if $H(s)$ analytic in the closed right half plane.

Periodic case: Nonlinear

$$\frac{dx(t)}{dt} = A(t)x(t), \quad T > 0 \text{ periodic.}$$

Periodic case: Nonlinear

$$\frac{dx(t)}{dt} = A(t)x(t), \quad T > 0 \text{ periodic.}$$

Definition

The origin of the system is exponentially stable if there exists γ, K positive such that:

$$\|x(t)\| \leq Ke^{-\gamma t} \|x(0)\|,$$

$x(\cdot)$ solution of the system.

Periodic case: Nonlinear

$$\frac{dx(t)}{dt} = A(t)x(t), \quad T > 0 \text{ periodic.}$$

Definition

The origin of the system is exponentially stable if there exists γ, K positive such that:

$$\|x(t)\| \leq K e^{-\gamma t} \|x(0)\|,$$

$x(\cdot)$ solution of the system.

Eigenvalues of $A(t)$ do not give the exponential stability of the origin anymore.

Periodic case: Monodromy operators

Periodic ODE Subsumed under **Floquet Theory**.

Periodic case: Monodromy operators

Periodic ODE Subsumed under **Floquet Theory**.

We introduce the fundamental solution $X(t, t_0)$ which satisfies :

$$\begin{aligned}\frac{d}{dt}X(t, t_0) &= A(t)X(t, t_0), \quad t \geq t_0, \\ X(t_0, t_0) &= Id, \text{ for all } t_0 \in \mathbb{R}.\end{aligned}$$

Definition (monodromy operators)

The monodromy operators of the system are the operator $X(T + t_0, t_0)$ for t_0 real.

Periodic case: Monodromy operators

Periodic ODE Subsumed under **Floquet Theory**.

We introduce the fundamental solution $X(t, t_0)$ which satisfies :

$$\begin{aligned}\frac{d}{dt}X(t, t_0) &= A(t)X(t, t_0), \quad t \geq t_0, \\ X(t_0, t_0) &= Id, \text{ for all } t_0 \in \mathbb{R}.\end{aligned}$$

Definition (monodromy operators)

The monodromy operators of the system are the operator $X(T + t_0, t_0)$ for t_0 real.

Theorem (classical)

Origin exp. stable if and only if the spectrum of the monodromy $X(T, 0)$ operator is strictly included in the unit disk.

Periodic case: Input-output system

Input-output system:

$$\begin{cases} \frac{d}{dt}y(t) = A(t)y(t) + B(t)u(t) \\ z(t) = C(t)y(t) + D(t)u(t), \quad t \geq 0, \end{cases}$$

Periodic case: Input-output system

Input-output system:

$$\begin{cases} \frac{d}{dt}y(t) = A(t)y(t) + B(t)u(t) \\ z(t) = C(t)y(t) + D(t)u(t), \quad t \in \mathbb{R} \end{cases}$$

Fourier series:

$$\begin{cases} \frac{d}{dt}y(t) = \sum_{k=-\infty}^{\infty} a_k e^{i\omega_0 k t} y(t) + \sum_{k=-\infty}^{\infty} b_k e^{i\omega_0 k t} u(t), \\ z(t) = \sum_{k=-\infty}^{\infty} c_k e^{i\omega_0 k t} y(t) + \sum_{k=-\infty}^{\infty} d_k e^{i\omega_0 k t} u(t), \end{cases}$$

$$\omega_0 := 2\pi/T.$$

Periodic case: Input-output system

Input-output system:

$$\begin{cases} \frac{d}{dt}y(t) = A(t)y(t) + B(t)u(t) \\ z(t) = C(t)y(t) + D(t)u(t), \quad t \geq 0, \end{cases}$$

Fourier series:

$$\begin{cases} \frac{d}{dt}y(t) = \sum_{k=-\infty}^{\infty} a_k e^{i\omega_0 k t} y(t) + \sum_{k=-\infty}^{\infty} b_k e^{i\omega_0 k t} u(t), \\ z(t) = \sum_{k=-\infty}^{\infty} c_k e^{i\omega_0 k t} y(t) + \sum_{k=-\infty}^{\infty} d_k e^{i\omega_0 k t} u(t), \end{cases}$$

$$\omega_0 := 2\pi/T.$$

Fourier series + Laplace transform Infinite time-invariant system.

Harmonic Transfer Function

$$Z(s) = ? \begin{pmatrix} \vdots \\ U(s + i\omega_0) \\ U(s) \\ U(s - i\omega_0) \\ \vdots \end{pmatrix}$$

Harmonic Transfer Function

There exists $H(\cdot)$ infinite matrix such that:

$$\begin{pmatrix} \vdots \\ Z(s + i\omega_0) \\ Z(s) \\ Z(s - i\omega_0) \\ \vdots \end{pmatrix} = H(s) \begin{pmatrix} \vdots \\ U(s + i\omega_0) \\ U(s) \\ U(s - i\omega_0) \\ \vdots \end{pmatrix}$$

Definition

The infinite matrix $H(s) := L_C [D_{\omega_0}(s) - L_A]^{-1} + L_D$ is called the **Harmonic Transfer Function (HTF)** matrix. (Wereley)

Where $D_{\omega_0}(s) := \text{Diag}(\dots, s + i\omega_0, s, s - i\omega_0, \dots)$ and
 $L_A := (a_{j-i})_{i,j} \mathbb{Z}$, $L_B := (b_{j-i})_{i,j} \mathbb{Z}$, $L_C := (c_{j-i})_{i,j} \mathbb{Z}$ and
 $L_D := (d_{j-i})_{i,j} \mathbb{Z}$.

HTF, singularities?

- HTF defines an operator valued analytic map
(values: continuous ops $l^2(Z)$ $l^2(Z)$)

HTF, singularities?

- HTF defines an operator valued analytic map
(values: continuous ops $l^2(Z) \rightarrow l^2(Z)$)
- | | *A priori* HTF can have many kind of singularities.

HTF, singularities?

- HTF defines an operator valued analytic map
(values: continuous ops $l^2(Z) \rightarrow l^2(Z)$)
- | | *A priori* HTF can have many kind of singularities.
 - its entries are matrix complex valued analytic maps

HTF, singularities?

- HTF defines an operator valued analytic map
(values: continuous ops $l^2(Z) \rightarrow l^2(Z)$)
- | | *A priori* HTF can have many kind of singularities.
 - its entries are matrix complex valued analytic maps
- | | Can be a singularity of the *HTF*, but what is the link with the singularities of the entries of the *HTF*?

Singularities HTF versus singularities of its entries

Let $H(s)$ an operator valued analytic map (values: continuous ops $l^2(Z) \rightarrow l^2(Z)$) on an open set S .

- $s_0 \in \overline{S}$ is a singularity of $H(\cdot)$ if $H(\cdot)$ does not admit an analytic continuation in s_0 . In particular if $H(s_0)$ is not a bounded operator from l^2 to l^2 then it is a singularity.

Singularities HTF versus singularities of its entries

Let $H(s)$ an operator valued analytic map (values: continuous ops $l^2(\mathbb{Z}) \rightarrow l^2(\mathbb{Z})$) on an open set S .

- $s_0 \in \overline{S}$ is a singularity of $H(\cdot)$ if $H(\cdot)$ does not admit an analytic continuation in s_0 . In particular if $H(s_0)$ is not a bounded operator from l^2 to l^2 then it is a singularity.
- However each coefficient of the infinite matrix can be defined in s_0 .

Singularities HTF versus singularities of its entries

Let $H(s)$ an operator valued analytic map (values: continuous ops $l^2(\mathbb{Z}) \rightarrow l^2(\mathbb{Z})$) on an open set S .

- $s_0 \in \overline{S}$ is a singularity of $H(\cdot)$ if $H(\cdot)$ does not admit an analytic continuation in s_0 . In particular if $H(s_0)$ is not a bounded operator from l^2 to l^2 then it is a singularity.
- However each coefficient of the infinite matrix can be defined in s_0 .

As example s_0 is an essential singularity:

$$\lim_{n \rightarrow +\infty} \|H(s_0)x_n\| = +\infty,$$

where $\|x_n\| = 1$ and all the coefficients of $H(s_0)$ exists and are finite.

Stability and HTF

Dwelling on **Floquet Theory**, we have:

Theorem (After Suarez and Quéré 02 or Louarroudi 14 for example)

Each element of the HTF is a meromorphic function. And its poles are included (possibly strictly) in the set of the complex numbers of the form $\frac{\ln(\zeta)+2i\pi k}{T}$ for all $k \in \mathbb{Z}$ and where ζ is an eigenvalue of the monodromy operator.

Theorem (classical)

Assume the system is observable and controllable, then for all ζ an eigenvalue of the monodromy operator there exists k integer such that $H(s)$, and one of its entries, has a pole in $\frac{\ln(\zeta)+2i\pi k}{T}$. Therefore origin exp. stable if and only if $H(s)$ analytic in the closed right half plane.

1 Introduction

- General context

2 Circuits without transmission lines

3 General circuits containing lossless transmission lines

1 Introduction

- General context

2 Circuits without transmission lines

3 General circuits containing lossless transmission lines

Lossless transmission lines

- Nonlinear circuit + lossless transmission lines

Lossless transmission lines

- Nonlinear circuit + lossless transmission lines
- Lossless transmission lines Telegrapher's equations (1-D hyperbolic) PDE Resolved form Delays equations.

Lossless transmission lines

- Nonlinear circuit + lossless transmission lines
- Lossless transmission lines Telegrapher's equations (1-D hyperbolic) PDE Resolved form Delays equations.
- Existence of a T -periodic solution.

Lossless transmission lines

- Nonlinear circuit + lossless transmission lines
- Lossless transmission lines Telegrapher's equations (1-D hyperbolic) PDE Resolved form Delays equations.
- Existence of a T -periodic solution.

Linearization around periodic solution:

$$\begin{cases} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^N B_{1,i}(t)y(t - \tau_i) \\ y(t) = \sum_{i=1}^N B_{2,i}(t)y(t - \tau_i) + A_2(t)x(t), \quad t \geq t_0. \end{cases}$$

square integrable and continuous solutions

For electronic applications: continuous solution and C^0 stability are needed.

However Fourier development of the system: L^2 solutions needed.

Anyway: L^2 stability is equivalent to the C^0 stability.

General linear system, T -periodic

$$\begin{cases} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^N B_{1,i}(t)y(t - \tau_i) \\ y(t) = \sum_{i=1}^N B_{2,i}(t)y(t - \tau_i) + A_2(t)x(t), \quad t \geq t_0, \end{cases}$$

General linear system, T -periodic

$$\begin{cases} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^N B_{1,i}(t)y(t - \tau_i) \\ y(t) = \sum_{i=1}^N B_{2,i}(t)y(t - \tau_i) + A_2(t)x(t), \quad t \geq t_0, \end{cases}$$

- State space: $YL^2 := \mathbb{R}^n \times L^2([- \tau_N, 0], \mathbb{R}^k)$.

General linear system, T -periodic

$$\begin{cases} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^N B_{1,i}(t)y(t - \tau_i) \\ y(t) = \sum_{i=1}^N B_{2,i}(t)y(t - \tau_i) + A_2(t)x(t), \quad t \geq t_0, \end{cases}$$

- State space: $YL^2 := \mathbb{R}^n \times L^2([- \tau_N, 0], \mathbb{R}^k)$.
- Solution operator $U(t, t_0) : YL^2 \rightarrow YL^2$

General linear system, T -periodic

$$\begin{cases} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^N B_{1,i}(t)y(t - \tau_i) \\ y(t) = \sum_{i=1}^N B_{2,i}(t)y(t - \tau_i) + A_2(t)x(t), \quad t \geq t_0, \end{cases}$$

- State space: $YL^2 := \mathbb{R}^n \times L^2([-\tau_N, 0], \mathbb{R}^k)$.
- Solution operator $U(t, t_0) : YL^2 \rightarrow YL^2$
- Monodromy operator $U(T, 0)$

General linear system, T -periodic

$$\begin{cases} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^N B_{1,i}(t)y(t - \tau_i) \\ y(t) = \sum_{i=1}^N B_{2,i}(t)y(t - \tau_i) + A_2(t)x(t), \quad t \geq t_0, \end{cases}$$

- State space: $YL^2 := \mathbb{R}^n \times L^2([-\tau_N, 0], \mathbb{R}^k)$.
- Solution operator $U(t, t_0) : YL^2 \rightarrow YL^2$
- Monodromy operator $U(T, 0)$

- $\left. \begin{array}{l} L^2 \text{ exponential} \\ \text{stability} \end{array} \right\} \left\{ \begin{array}{l} Sp(U(T, 0)) \text{ included in} \\ \text{disc of radius } r < 1 \end{array} \right.$

General linear system, T -periodic

$$\left\{ \begin{array}{l} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^N B_{1,i}(t)y(t - \tau_i) \\ y(t) = \sum_{i=1}^N B_{2,i}(t)y(t - \tau_i) + A_2(t)x(t), \quad t \geq t_0, \end{array} \right.$$

- State space: $YL^2 := \mathbb{R}^n \times L^2([- \tau_N, 0], \mathbb{R}^k)$.
- Solution operator $U(t, t_0) : YL^2 \rightarrow YL^2$
- Monodromy operator $U(T, 0)$
- L^2 exponential stability $\left. \vphantom{\begin{array}{l} L^2 \text{ exponential} \\ \text{stability} \end{array}} \right\} \left\{ \begin{array}{l} Sp(U(T, 0)) \text{ included in} \\ \text{disc of radius } r < 1 \end{array} \right.$

Behaviour at high frequency

High frequency limit system :

$$\begin{cases} x(t) = 0 \\ y(t) = \sum_{i=1}^N B_{2,i}(t)y(t - \tau_i), \quad t \geq t_0, \end{cases}$$

- State space: $Y\tilde{L}^2 := \{0_n\} \times L^2([- \tau_N, 0], \mathbb{R}^k)$.
- Solution operator $V(t, t_0) : Y\tilde{L}^2 \rightarrow Y\tilde{L}^2$.
- Monodromy operator $V(T, 0)$.

■

$$\left. \begin{array}{l} L^2 \text{ exponential} \\ \text{stability} \end{array} \right\} \left\{ \begin{array}{l} Sp(V(T, 0)) \text{ included in} \\ \text{disc of radius } r < 1 \end{array} \right.$$

Compact perturbation

Lemma

We have :

$$U(t, t_0) = V(t, t_0)P + K(t, t_0), \quad t \geq t_0$$

with $K(t, t_0)$ compact operator $\gamma L^2 \rightarrow \gamma L^2$ for all t, t_0 and P the canonical projection $\gamma L^2 \rightarrow \gamma \tilde{L}^2$.

Compact perturbation

Lemma

We have :

$$U(t, t_0) = V(t, t_0)P + K(t, t_0), \quad t \geq t_0$$

with $K(t, t_0)$ compact operator $\gamma L^2 \rightarrow \gamma L^2$ for all t, t_0 and P the canonical projection $\gamma L^2 \rightarrow \gamma \tilde{L}^2$.

Theorem (classical)

A compact perturbation does not change the essential spectrum and it can add only point spectrum, i.e. eigenvalues.

Theorem

If the high frequency system is L^2 exponentially stable then the monodromy operator $U(T, 0)$ possesses at most a finite number of eigenvalues ζ_1, \dots, ζ_n outside a disk of a radius strictly less than 1.

Theorem

If the high frequency system is L^2 exponentially stable then the monodromy operator $U(T, 0)$ possesses at most a finite number of eigenvalues ζ_1, \dots, ζ_n outside a disk of a radius strictly less than 1.

High frequency system dissipates energy

Theorem

If the high frequency system is L^2 exponentially stable then the monodromy operator $U(T, 0)$ possesses at most a finite number of eigenvalues ζ_1, \dots, ζ_n outside a disk of a radius strictly less than 1.

High frequency system dissipates energy
(BFLP21) L^2 exp. stability of the high frequency system. It is a reasonable assumption for electric circuits.

Theorem

If the high frequency system is L^2 exponentially stable then the monodromy operator $U(T, 0)$ possesses at most a finite number of eigenvalues ζ_1, \dots, ζ_n outside a disk of a radius strictly less than 1.

High frequency system dissipates energy
(BFLP21) L^2 exp. stability of the high frequency system. It is a reasonable assumption for electric circuits.

Instability of the general linear system is governed just by a finite number of eigenvalues.

Input-Output system

$$\left\{ \begin{array}{l} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^N B_i^1(t)y(t - \tau_i) + C_1(t)u(t) \\ y(t) = \sum_{i=1}^N B_i^2(t)y(t - \tau_i) + A_2(t)x(t) + C_2(t)u(t) \\ z(t) = \sum_{i=1}^N B_i^3(t)y(t - \tau_i) + A_3(t)x(t) + C_3(t)u(t), \quad t \geq 0, \end{array} \right.$$

- $x(t), y(t), z(t) = 0$ for $t < 0$,
- Input $u \in L_{loc}^2([0, +\infty), \mathbb{R})$ current perturbation, output z the voltage,
- All coefficients are T -periodic.

Harmonic Transfer Function

$$\begin{pmatrix} \vdots \\ Z(s + i\omega_0) \\ Z(s) \\ Z(s - i\omega_0) \\ \vdots \end{pmatrix} = H(s) \begin{pmatrix} \vdots \\ U(s + i\omega_0) \\ U(s) \\ U(s - i\omega_0) \\ \vdots \end{pmatrix}$$

- HTF is an operator valued analytic map
(values: continuous ops $l^2(\mathbb{Z}) \rightarrow l^2(\mathbb{Z})$)
- its entries are complex valued analytic maps

Structure of the Harmonic Transfer Function

Define $z_{j,k} = \frac{\ln(\zeta_j) + 2ik\pi}{T}$ for j in $\{1 \dots n\}$, k in \mathbb{Z} .

Fundamental structure HTF for high dissipative amplifiers

Theorem

In the half space $\{s \in \mathbb{C}, \operatorname{Re}(s) < \gamma\}$ for some $\gamma < 0$,

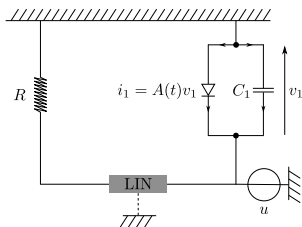
- *H is a meromorphic operator $\ell^2(\mathbb{Z}) \rightarrow \ell^2(\mathbb{Z})$ whose poles are contained in $\{z_{j,k}, j \in \{1 \dots n\}, k \in \mathbb{Z}\}$.*
- *Under observability/controllability assumptions, for all j , there is at least a k such that $z_{j,k}$ is one pole of H , and also a pole of one entries of the HTF.*

If no entry of the HTF has poles in right half plane, we have the exponential C^0 stability.

Singularities of the HTF in the left right half plane?

- High frequency dissipative circuit HTF meromorphic right half plane.
- Rational tools sometimes performs rational approximation on all the complex plane.
- Needs to know if there is just poles in the left half plane?

Simple example (Brayton 1976)



Input: u

output: $Z = v_1$

Theorem

If $T/\tau_1 \notin \mathbb{Q}$ and the system is observable, all points of a certain vertical line in the left-half plane are essential singularities of the HTF, as an operator valued analytic map.

But are they singularities for some entries of the HTF ?

Thank You!